**Title: Some Results on Degenerate Elliptic Equations**

**Lecturer: Prof Hua Chen (Wuhan University)**

**Duration: 8 hours**

## Overview

This is an extension course on elliptic PDE for the PhD students after they have already learned the fundamental courses on PDEs. The course will include some fundamental results on the degenerate elliptic equations, i.e. hypoellipticity, Hörmander's sum of square theorem, Bony's maximum principle, sub-elliptic estimate and sub-elliptic metric, logarithmic regularity estimate and logarithmic Sobolev inequality, estimates of eigenvalues for finitely degenerate and infinitely degenerate elliptic operators, and the boundary-value problems for linear and nonlinear degenerate elliptic equations.

## Learning Outcomes

After learning this course the students may know some fundamental results on the field of degenerate elliptic equations and understand the basic idea and technique on treatment of such kind of degenerate PDEs.

## Synopsis

Lecture 1: Definition of vector fields, sum of square operator, hypoellipticity, commutator technique, Hörmander's Sum of Square Theorem.

Lecture 2：Sharp sub-elliptic Estimate, sub-elliptic metric and doubling property.

Lecture 3：Weighted Sobolev Spaces and Holder Spaces, corresponding Sobolev embedding theorems, Bony's maximum principle.

Lecture 4：Boundary-value problems for linear and nonlinear degenerate elliptic equations.

Lecture 5：Estimates of eigenvalues for finitely degenerate elliptic operator, Fourier method.

Lectuer 6：Infinitely degenerate elliptic equations, logarithmic regularity estimate and hypoellipticity.

Lecture 7：Logarithmic Sobolev inequality, Hardy type inequality, boundary-value problems for nonlinear infinitely degenerate elliptic equations.

Lecture 8：Estimates of eigenvalues for infinitely degenerate elliptic operators.

## Prerequisites

Prerequisites include knowing the contents of courses of “Introduction to PDE” and “Analysis of PDEs”.

## Core Reading

L.-C. Evans, Partial Differential Equations, American Mathematical Society, 2nd edition, 2010.

G. Folland, Introduction to Partial Differential Equations, 2nd edition, Princeton University Press, 1995.

M. Struwe, Variational Methods, Applications to Nonlinear Partial Differential Equations and Hamiltonian Systems, Fourth edition, Springer-Verlag, 2008.

S. Levendorskii, Degenerate Elliptic Equations, Kluwer Academic Publishers, 2010.

## Further Reading

1). Bony, J. M., Principe du maximum, inegalite de Harnack et unicite du probleme de Cauchy pour les operateurs elliptiques degenerees, Ann. Inst. Fourier 19 (1969), 227-304.

2). Christ, M., Hypoellipticity in the infinitely degenerate regime, Proceedings of the International Conference on Several Complex Variable at the Ohio State University, USA (1997).

3). Fefferman, C., Phong, D., Subelliptic eigenvalue problems, Proceedings of the Conference on Harmonic Analysis in Honor of Antoni Zygmund, Wadsworth Math. Series, pp. 590–606 (1981).

4). Hajlasz, P., Koskela, P., Sobolev met Poincaré, Mem. Am. Math. Soc. 145, 1–101 (2000).

5). Hörmander, L., Hypoelliptic second order differential equations, Acta Math. 119, 147–171 (1967).

6). Kohn, J.J., Subellipticity of $\bar{\partial}$ -Neumann of problem on pseudoconvex domains, sufficient conditions, Acta Math. 142, 79–122 (1979).

7). Li, P., Yau, S.T., On the Schrödinger equation and the eigenvalue problem, Commun. Math. Phys. 88, 309-318 (1983).

8). Métivier, G., Fonction spectrale d’opérateurs non elliptiques, Commun. PDE. 1, 467–519 (1976).

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9). Morimoto, Y., Xu, C.J., Logarithmic Sobolev inequality and semi-linear Dirichlet problems for infinitely degenerate elliptic operators, Astérisque 234, 245–264 (2003).

10). Nagel, A., Stein, E.M., Wainger, E.M.S., Balls and metrics defined by vector fields I, basic properties, Acta Math. 155, 103–147 (1985).

11). Rothschild, L., and Stein, E. M., Hypoelliptic differential operators and nilpotent Lie groups, Acta Math. 137 (1977), 247-320.

12). Xu, C.J., Regularity problem for quasi-linear second order subelliptic equations, Commun. Pure Appl. Math. 45, 77–96 (1992).

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